

Hawking radiation of charged particles as tunneling from Reissner-Nordström-de Sitter black holes with a global monopole

Qing-Quan Jiang

*College of Physical Science and Technology, Central China Normal University,
Wuhan, Hubei 430079, People's Republic of China
Institute of Theoretical Physics, China West Normal University, Nanchong,
Sichuan 637002, People's Republic of China*

Shuang-Qing Wu

*College of Physical Science and Technology, Central China Normal University,
Wuhan, Hubei 430079, People's Republic of China*

Abstract

Applying Parikh's semi-classical tunneling method, we consider Hawking radiation of the charged massive particles as a tunneling process from the Reissner-Nordström-de Sitter black hole with a global monopole. The result shows that the tunneling rate is related to the change of Bekenstein-Hawking entropy and the radiant spectrum is not a pure thermal one, but is consistent with an underlying unitary theory.

Key words: Charged particle; Radiation; Tunneling rate; Bekenstein-Hawking entropy; Energy conservation and charge conservation

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Email address: sqwu@phy.ccnu.edu.cn, Corresponding author (Shuang-Qing Wu).

1 Introduction

In 1974, Hawking [1] proved that the black hole can emit particles from its event horizon with a temperature proportional to its surface gravity, and the radiant spectrum is a pure thermal one, which implies the loss of information of black hole after it has evaporated away and disappeared completely [2]. Though a complete resolution of the information loss paradox must be in the framework of quantum gravity and/or the unitary theory of string/M-theory, Hawking argued that the information could come out if the outgoing radiation were not exactly thermal but had subtle corrections.

Recently, Parikh and Wilczek [3] put forward a semi-classical tunnelling method to investigate Hawking radiation of the static Schwarzschild and Reissner-Nordström black holes, they found that the radiant spectrum of the black hole is not a pure thermal one and the derived tunneling rate is related to the change of Bekenstein-Hawking entropy. In their methodology, Hawking radiation is treated as a tunneling process with the tunneling potential barrier produced by the outgoing particle itself. The key trick to calculate the tunneling rate is to find a coordinate system well-behaved at the event horizon. However, this method is currently limited to discuss the tunneling rate of the uncharged massless particles only [3,4,5]. For black holes with a charge, the emitted outgoing particles can be charged also, not only should the energy conservation but also the charge conservation be considered [6].

On the other hand, researches on the charged black hole with a positive cosmological constant and with a global monopole become important due to the following reasons: (1) The recent observed accelerating expansion of our universe indicates the cosmological constant might be a positive one [7]; (2) Conjecture about de Sitter/conformal field theory (CFT) correspondence [8]; (3) There might exist topological defects in the early universe [9]; etc.

Combined with the reasons mentioned above, in this Letter we extend the Parikh's method to investigate the Hawking radiation of the charged particle via tunneling from the Reissner-Nordström-de Sitter black hole with a global monopole whose Arnowitt-Deser-Misner (ADM) mass is $(1 - 8\pi\eta^2)$ times than that of mass parameter. Our result shows that the emission rate of the charged particle is connected with the Bekenstein-Hawking entropy, and the corrected radiant spectrum is not a pure thermal one, but is consistent with an underlying unitary theory.

Our Letter is outlined as follows: In Section 2, we introduce the generalized Painlevé coordinate transformation and present the radial geodesic equation of charged particles. In Sections 3 and 4, we investigate Hawking radiation as tunneling from the event horizon and the cosmological horizon, and compute

the tunneling rate from these two horizons, respectively. Finally we give some discussions about our results.

2 Generalized Painlevé coordinate transformation and the radial geodesics of charged particles

The line element of a Reissner-Nordström-de Sitter black hole with a global monopole is [10]

$$ds^2 = -\Delta dt_R^2 + \Delta^{-1} dr^2 + (1 - 8\pi\eta^2)r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $\Delta = 1 - 2M/r + Q^2/r^2 - (\Lambda/3)r^2$, η is a symmetry breaking constant related to the global monopole, M is the mass parameter, Q is the charge of the black hole, Λ is a positive cosmological constant, and t_R is the coordinate time for the black hole. In general, the black hole has an inner horizon (IH), an event horizon (EH) and a cosmological horizon (CH), all of them satisfying the horizon equation $\Delta = 0$. In this Letter we shall consider the most general case where neither of these horizons coincides with the other one.

To remove the coordinate singularity in the metric (1), we introduce a generalized Painlevé coordinate transformation

$$dt_R = dt \mp \frac{\sqrt{1-\Delta}}{\Delta} dr, \quad (2)$$

and obtain the Painlevé-like line element of the Reissner-Nordström-de Sitter black hole with a global monopole as follows

$$ds^2 = -\Delta dt^2 \pm 2\sqrt{1-\Delta} dt dr + dr^2 + (1 - 8\pi\eta^2)r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

where a plus (minus) sign denotes the space-time line element of the charged massive outgoing (ingoing) particles at the EH (CH). In Eq. (3), the Painlevé-like coordinate system has many attractive features. First, the metric is well behaved at the EH and CH; Secondly, it satisfies Landau's condition of the coordinate clock synchronization; Thirdly, the new form of the line element is stationary, but not static. These characters are useful to investigate the tunneling radiation of the charged massive particles across the horizons.

It should be pointed out that unlike the asymptotically flat case, the Painlevé-like coordinate for the asymptotically non-flat space-time is not unique. In fact, there is another form for the metric (1)

$$ds^2 = -\Delta dt^2 \pm 2\sqrt{1-\Delta/(1-\Lambda r^2/3)} dt dr + (1-\Lambda r^2/3)^{-1} dr^2 + (1-8\pi\eta^2)r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

which approaches to the de Sitter space in the vacuum case where $\eta = 0$.

Now, let us work with the metric in the new form (3) and obtain the radial geodesics of the charged massive particles, which is different from that of the uncharged massless particles that follow the radial null geodesics

$$\dot{r} = \frac{dr}{dt} = \pm 1 \mp \sqrt{1 - \Delta}. \quad (4)$$

According to de Broglie's hypothesis, from the definition of the phase velocity v_p and the group velocity v_g , we have

$$v_p = \frac{1}{2}v_g. \quad (5)$$

Since the tunneling process is an instantaneous effect, the metric in the line element (3) satisfies Landau's condition of the coordinate clock synchronization, the coordinate time difference of two events, which take place simultaneously in different places, is

$$dt = -\frac{g_{tr}}{g_{tt}}dr_c, \quad (d\theta = d\phi = 0), \quad (6)$$

where dr_c is the location of the tunneling particle. So the group velocity can be expressed as

$$v_g = \frac{dr_c}{dt} = -\frac{g_{tt}}{g_{tr}} = \pm \frac{r^2 - 2Mr + Q^2 - (\Lambda/3)r^4}{\sqrt{2Mr^3 - Q^2r^2 + (\Lambda/3)r^6}}, \quad (7)$$

therefore the phase velocity (the radial geodesics) is

$$\dot{r} = v_p = -\frac{g_{tt}}{2g_{tr}} = \pm \frac{r^2 - 2Mr + Q^2 - (\Lambda/3)r^4}{2\sqrt{2Mr^3 - Q^2r^2 + (\Lambda/3)r^6}}, \quad (8)$$

where $+$ ($-$) sign denotes the phase velocity of the charged particles tunneling across the EH (CH). During the process of a charged massive particle tunneling across the potential barrier, the self-interaction effect of the electro-magnetic field on the emitted particles should not be ignored, and the temporal component of electro-magnetic potential is

$$A_t = \pm \frac{Q}{r}. \quad (9)$$

In the remaining two sections, we shall discuss Hawking radiation from the event horizon and the cosmological horizon, respectively, and calculate the tunneling rate from each horizon. Since the overall picture of tunneling radiation for the metric is very involved, to simplify the discussion we will consider

the outgoing radiation from the EH, and ignore the incoming radiation from the CH, for the moment when we deal with the black hole event horizon. While dealing with the CH case, we shall only consider the incoming radiation from the CH and ignore the outgoing radiation from the EH.

3 Tunneling rate of charged particles at the EH

According to the energy conservation and the charge conservation, one can assume that the total ADM mass and charge of the hole-particle system are held fixed whereas the mass and the charge of the hole are allowed to fluctuate, the black hole mass and charge will become $M - \omega$, $Q - q$ when a particle with energy ω and charge q has evaporated from the EH. Considering the charged particle tunnels out from the EH along the radial direction, we can get the new line element of the black hole in the EH case

$$ds^2 = -\Delta' dt^2 + 2\sqrt{1 - \Delta'} dt dr + dr^2 + (1 - 8\pi\eta^2)r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (10)$$

where $\Delta' = 1 - 2(M - \omega)/r + (Q - q)^2/r^2 - (\Lambda/3)r^2$. Accordingly the radial geodesics of the charged massive particles tunneling out from the EH is

$$\dot{r} = \frac{r^2 - 2(M - \omega)r + (Q - q)^2 - (\Lambda/3)r^4}{2\sqrt{2(M - \omega)r^3 - (Q - q)^2r^2 + (\Lambda/3)r^6}}, \quad (11)$$

and the non-zero component of electro-magnetic potential becomes

$$A_t = \frac{Q - q}{r}. \quad (12)$$

When the charged particle tunnels out, the effect of the electro-magnetic field should be taken into account. So the matter-gravity system consists of the black hole and the electro-magnetic field outside the hole. As the Lagrangian function of the electro-magnetic field corresponding to the generalized coordinates described by A_μ is $-(1/4)F_{\mu\nu}F^{\mu\nu}$, we can find that the generalized coordinate $A_\mu = (A_t, 0, 0, 0)$ is an ignorable coordinate. In order to eliminate the degree of freedom corresponding to A_μ , the imaginary part of the action for the charged massive particle should be written as

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{t_i}^{t_f} (L - P_{A_t} \dot{A}_t) dt = \text{Im} \int_{r_{ie}}^{r_{fe}} (P_r \dot{r} - P_{A_t} \dot{A}_t) \frac{dr}{\dot{r}} \\ &= \text{Im} \int_{r_{ie}}^{r_{fe}} \left[\int_{(0, 0)}^{(P_r, P_{A_t})} (\dot{r} dP'_r - \dot{A}_t dP'_{A_t}) \right] \frac{dr}{\dot{r}}, \end{aligned} \quad (13)$$

where r_{ie} and r_{fe} represent the locations of the EH before and after the particle with energy ω and charge q tunnels out. According to Hamilton's canonical equation of motion, we have

$$\begin{aligned}\dot{r} &= \left. \frac{dH}{dP_r} \right|_{(r; A_t, P_{A_t})}, & dH|_{(r; A_t, P_{A_t})} &= (1 - 8\pi\eta^2)d(M - \omega), \\ \dot{A}_t &= \left. \frac{dH}{dP_{A_t}} \right|_{(A_t; r, P_r)}, & dH|_{(A_t; r, P_r)} &= (1 - 8\pi\eta^2)\frac{Q - q}{r}d(Q - q),\end{aligned}\quad (14)$$

where ω and q are the energy and the charge of the emitted particle. Because of the existence of a global monopole in the black hole background, the total ADM mass and the total charge in the EH case are $M_\infty = (1 - 8\pi\eta^2)M$ [11] and $Q_\infty = (1 - 8\pi\eta^2)Q$, respectively. [For the sake of convenience, we take the mass and charge of the particle measured at infinity as $\omega_\infty = (1 - 8\pi\eta^2)\omega$ and $q_\infty = (1 - 8\pi\eta^2)q$.] Eq. (14) represents the energy change of the hole because of the loss of mass and charge when a particle tunnels out. Substituting Eqs. (11) and (14) into Eq. (13) and switching the order of integral, we obtain

$$\begin{aligned}\text{Im}S &= \text{Im} \int_{r_{ie}}^{r_{fe}} \int_{(1-8\pi\eta^2)(M, Q)}^{(1-8\pi\eta^2)(M-\omega, Q-q)} \left[dH|_{(r; A_t, P_{A_t})} - dH|_{(A_t; r, P_r)} \right] \frac{dr}{\dot{r}} \\ &= \text{Im} \int_{(1-8\pi\eta^2)(M, Q)}^{(1-8\pi\eta^2)(M-\omega, Q-q)} \int_{r_{ie}}^{r_{fe}} \frac{2\sqrt{2(M-\omega')r^3 - (Q-q')^2r^2 + (\Lambda/3)r^6}}{r^2 - 2(M-\omega')r + (Q-q')^2 - (\Lambda/3)r^4} \\ &\quad \times (1 - 8\pi\eta^2) \left[d(M - \omega') - \frac{Q - q'}{r}d(Q - q') \right] dr.\end{aligned}\quad (15)$$

Since $1 - 2(M - \omega')/r + (Q - q')^2/r^2 - (\Lambda/3)r^2 = 0$ satisfies the horizon equation after the particle with energy ω' and charge q' tunnels out, there exists a single pole in Eq. (15). Let us carry out the integral by deforming the contour around the pole so as to ensure that the positive energy solutions decay in time, and get

$$\text{Im}S = -(1 - 8\pi\eta^2)\text{Im} \int_{r_{ie}}^{r_{fe}} (i\pi r)dr = -\frac{\pi}{2}(1 - 8\pi\eta^2)(r_{fe}^2 - r_{ie}^2). \quad (16)$$

So the relationship between the tunneling rate and the imaginary part of the particle's action is

$$\Gamma \sim e^{-2\text{Im}S} = e^{\pi(1-8\pi\eta^2)(r_{fe}^2 - r_{ie}^2)} = e^{(A_{fe} - A_{ie})/4} = e^{\Delta S_{EH}}, \quad (17)$$

where A_{ie} and A_{fe} denote the event horizon area before and after the charged particle tunnels out, and ΔS_{EH} is the change of Bekenstein-Hawking entropy. From Eq. (17), we find that the tunneling rate at the EH is related to the

Bekenstein-Hawking entropy, and is consistent with an underlying unitary theory.

4 Tunneling rate of charged particles at the CH

In this section, we will discuss the Hawking radiation of the charged particle via tunneling at the CH. Different from the particle's tunneling behavior in the EH case discussed in the last section, the particle is found to tunnel into the CH. So when the particle with energy ω and charge q tunnels into the CH, we can get the new line element as follows

$$ds^2 = -\Delta'' dt^2 - 2\sqrt{1 - \Delta''} dt dr + dr^2 + (1 - 8\pi\eta^2)r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (18)$$

where $\Delta'' = 1 - 2(M + \omega)/r + (Q + q)^2/r^2 - (\Lambda/3)r^2$. Using the same method, the phase velocity (the radial geodesics) of the charged particle tunneling into the CH can be expressed as

$$\dot{r} = -\frac{r^2 - 2(M + \omega)r + (Q + q)^2 - (\Lambda/3)r^4}{2\sqrt{2(M + \omega)r^3 - (Q + q)^2r^2 + (\Lambda/3)r^6}}, \quad (19)$$

and the electro-magnetic potential becomes accordingly as

$$A_t = -\frac{Q + q}{r}. \quad (20)$$

According to Hamilton's canonical equation of motion, when a particle with energy ω and charge q is absorbed by the CH of the black hole, we can get

$$\begin{aligned} dH|_{(r; A_t, P_{A_t})} &= -(1 - 8\pi\eta^2)d(M + \omega), \\ dH|_{(A_t; r, P_r)} &= -(1 - 8\pi\eta^2)\frac{Q + q}{r}d(Q + q), \end{aligned} \quad (21)$$

where ω and q are the energy and the charge of the absorbed particle. Due to the presence of a global monopole in the black hole background, the total ADM mass and the total charge in the CH case are $M_\infty = -(1 - 8\pi\eta^2)M$ and $Q_\infty = -(1 - 8\pi\eta^2)Q$, respectively. In the same way, the imaginary part of the action for the charged massive particle incoming from the CH can be expressed as

$$\begin{aligned}
\text{Im}S &= \text{Im} \int_{t_i}^{t_f} (L - P_{A_t} \dot{A}_t) dt = \text{Im} \int_{r_{ic}}^{r_{fc}} (P_r \dot{r} - P_{A_t} \dot{A}_t) \frac{dr}{\dot{r}} \\
&= \text{Im} \int_{-(1-8\pi\eta^2)(M, Q)}^{-(1-8\pi\eta^2)(M+\omega, Q+q)} \int_{r_{ic}}^{r_{fc}} \frac{2\sqrt{2(M+\omega')r^3 - (Q+q')^2r^2 + (\Lambda/3)r^6}}{r^2 - 2(M+\omega')r + (Q+q')^2 - (\Lambda/3)r^4} \\
&\quad \times (1 - 8\pi\eta^2) \left[d(M+\omega') - \frac{Q+q'}{r} d(Q+q') \right] dr. \tag{22}
\end{aligned}$$

In Eq. (22), r_{ic} and r_{fc} are the locations of the CH before and after the particle with energy ω and charge q is absorbed by the CH, and we find that $1 - 2(M+\omega')/r + (Q+q')^2/r^2 - (\Lambda/3)r^2 = 0$ is the horizon equation after the particle tunnels into the CH, so there exists a single pole in Eq. (22). Deforming the contour around the pole and carrying out the integral, we have

$$\text{Im}S = -(1 - 8\pi\eta^2) \text{Im} \int_{r_{ic}}^{r_{fc}} (i\pi r) dr = -\frac{\pi}{2} (1 - 8\pi\eta^2) (r_{fc}^2 - r_{ic}^2). \tag{23}$$

So the tunneling rate at the CH is

$$\Gamma \sim e^{-2\text{Im}S} = e^{\pi(1-8\pi\eta^2)(r_{fc}^2 - r_{ic}^2)} = e^{(A_{fc} - A_{ic})/4} = e^{\Delta S_{CH}}, \tag{24}$$

where A_{ic} and A_{fc} are the cosmological horizon area before and after the charged massive particle tunnels into the CH, and ΔS_{CH} is the change of the Bekenstein-Hawking entropy at the CH. From Eq. (24), we learn that tunneling rate at the CH of the Reissner-Nordström-de Sitter black hole with a global monopole is connected with Bekenstein-Hawking entropy.

5 Summary and Discussions

In summary, we find that when the charged massive particle tunnels across the event horizon (EH) and the cosmological horizon (CH) of a Reissner-Nordström-de Sitter black hole with a global monopole, the radiant spectrum is not a pure thermal one, the tunneling rate is related to the change of Bekenstein-Hawking entropy corresponding to each horizon, and is consistent with an underlying unitary theory. So the Hawking radiation can be viewed as an ideal case only, it is possible for a not precisely thermal radiation to carry out information during the radiation process of the black holes, and the underlying unitary theory is reliable. The result obtained in this paper provides further evidence to support the Parikh's tunneling picture, which might serve as a mechanism to deal with the information loss paradox.

We would like to point out that a large class of previous results existed in the literature can be enclosed as special case of ours obtained here. In particular,

results obtained in Ref. [3,4] can be recovered. For example, in the case where $\Lambda = 0$ and $\eta = 0$, the Reissner-Nordström-de Sitter black hole with a global monopole reduces to the Reissner-Nordström black hole. Considering an uncharged massless particle but with energy ω tunnels across the event horizon, we know that $r_i = M + \sqrt{M^2 - Q^2}$ and $r_f = M - \omega + \sqrt{(M - \omega)^2 - Q^2}$ are the horizons of the black hole before and after the emission of the particle. According to Eq. (17), the tunneling rate is

$$\Gamma \sim e^{-2\text{Im}S} = e^{2\pi \left[(M-\omega)^2 + (M-\omega)\sqrt{(M-\omega)^2 - Q^2} - M^2 - M\sqrt{M^2 - Q^2} \right]} = e^{\Delta S_{BH}}, \quad (25)$$

which is same one as that obtained in Ref. [3].

For another special case when $\Lambda = 0$, $Q = 0$, and $\eta = 0$, the black hole metric considered here reduces to the Schwarzschild black hole, one can derive the event horizon of the black hole before and after a particle with energy ω is emitted, namely, $r_i = 2M$ and $r_f = 2(M - \omega)$. According to Eq. (17), the tunneling rate at the event horizon will be reduced to

$$\Gamma \sim e^{-2\text{Im}S} = e^{-8\pi(M-\omega/2)} = e^{\Delta S_{BH}}, \quad (26)$$

which coincides with Parikh's result in the Schwarzschild black hole case.

In addition, our discussions made here can be directly extended to the anti-de Sitter case [12] by changing the sign of the cosmological constant to a negative one, and also can be easily generalized to higher dimensional spherically symmetric black holes case.

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References

- [1] S.W. Hawking, Nature **248** (1974) 30;
S.W. Hawking, Commun. Math. Phys. **43** (1975) 199.
- [2] S.W. Hawking, Phys. Rev. D **72** (2005) 084013.
- [3] M.K. Parikh, F. Wilczek, Phys. Rev. Lett. **85** (2000) 5042;
M.K. Parikh, Int. J. Mod. Phys. D **13** (2004) 2351;
M.K. Parikh, Phys. Lett. B **546** (2002) 189.

- [4] A.J.M. Medved, Phys. Rev. D **66** (2002) 124009;
S. Hemming, E. Keski-Vakkuri, Phys. Rev. D **64** (2001) 044006;
E.C. Vagenas, Phys. Lett. B **559** (2003) 65;
E.C. Vagenas, Phys. Lett. B **533** (2002) 302;
A.J.M. Medved, Class. Quantum Grav. **19** (2002) 589;
A.J.M. Medved, E.C. Vagenas, Mod. Phys. Lett. A **20** (2005) 1723;
A.J.M. Medved, E.C. Vagenas, Mod. Phys. Lett. A **20** (2005) 2449;
M. Arzano, A.J.M. Medved, E.C. Vagenas, JHEP **09** (2005) 037;
M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, JHEP **05** (2005) 014;
S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, Class. Quantum Grav. **19** (2002) 2671;
T. Padmanabhan, Mod. Phys. Lett. A **19** (2004) 2637.
- [5] S.Z. Yang, Q.Q. Jiang, H.L. Li, Chin. Phys. **14** (2005) 2411.
- [6] P. Kraus, F. Wilczek, Nucl. Phys. B **437** (1995) 231;
J.Y. Zhang, Z. Zhao, JHEP **10** (2005) 055.
- [7] A.G. Reiss *et al.*, Astron. J. **116** (1998) 1009;
S. Perlmutter *et al.*, Astrophys. J. **517** (1999) 565;
J.P. Ostriker, P.J. Steinhardt, Nature **377** (1995) 600.
- [8] A. Strominger, JHEP **10** (2001) 034;
D. Klemm, Nucl. Phys. B **625** (2002) 295.
- [9] T. Tamaki, N. Sakai, Phys. Rev. D **69** (2004) 044018;
M. Yamaguchi, Phys. Rev. D **64** (2001) 081301.
- [10] C.J. Gao, Y.G. Shen, Chin. Phys. Lett. **19** (2002) 477.
- [11] B. Jensen, Phys. Lett. B **334** (1994) 309;
S. Bose, N. Dadhich, Phys. Rev. D **60** (1999) 064010;
U. Nucamendi, D. Sudarsky, Class. Quantum Grav. **14** (1997) 1309.
- [12] X.Z. Li, J.G. Hao, Phys. Rev. D **66** (2002) 107701;
B. Bertrand, Y. Brihaye, B. Hartmann, Class. Quantum Grav. **20** (2003) 4173.